Effects of Multiple CW Interferers on Carrier Synchronization*

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Abstract—**In this paper the modeling and analysis of of effects multiple CW interferers on Phase Locked Loop performance is conducted such that the operating vulnerability to jamming, interference or multipath can be accessed. First, in the absence of additive noise and presence of interference within the loop passband, a statistical model is developed which demonstrates the effect of multiple CW interferers on the phase error process. The steady state phase error is characterized statistically through its probability density function and moments. The conditions for keeping locked and the probability that the loop stays locked to the desired carrier frequency are derived for arbitrary loop filter characteristics. A novel outage criterion is established for the satisfactory operation of the loop and outage probabilities are presented for certain operating scenarios. Second, assuming the presence of Additive White Gaussian Noise, a Fokker Planck analysis is conducted in order to obtain the probability density function, moments and cycle slip rates of the phase error process.**

I. INTRODUCTION

Analysis of carrier synchronization loop performance in the presence of a single CW interferer has been the subject of many studies in the literature. Several of these elaborate on performance characterization with the assumption of absence of noise effecting the system. [1], [2], [3], [4], [5], [6], [7], [8], [9], [10]. Some articles take the presence of noise into account [11], [12], [13], [14].

In [15] results of previous literature on the performance of Phase Locked Loops (PLL) in the presence of a CW interferer are generalized to extend their practical applicability. Namely, in [15], the assumption of no initial detuning between the desired carrier and the quiescent Voltage Controlled Oscillator (VCO) frequencies is uplifted in the analysis for the cases of both the presence and absence of noise. Also in the presence of noise the earlier assumption that the interferer and the desired carrier are of same frequency [13], [14] is not used.

Examination of effects of multiple CW interferers on PLLs is of practical intererest in several applications. Deep space coherent communications an tracking systems utilize a tone to convey the signal carrier phase which is tracked by a PLL which may be effected by CW radio frequency interference [16].

Frequently, the local oscillator (LO) output used for down conversion in a communications receiver possesses spurious frequency components near the LO center frequency. When such LO's are used to down convert the input signal, the resulting output contains spurious components which are subjected to the loop input. Such components degrade the loop performance. The results of this study can be used to identify the vulnerability PLLs to spurious signals and specify the acceptable level of spurious terms.

Monopulse angle tracking radars are being widely used for tracking airborne vehicles in which the receivers often use PLLs for coherent detection of the received signals [2], [3], [4]. Due to high mobility of the vessels, the radar signatures of a set of targets span a large bandwidth due to Doppler shifts, and, each signature may also be expected to have a large Doppler rate. Hence, the PLL's used in the tracking radars used should have large front end bandwidths [19]. Thus many signals from several targets are not uncommon in such scenarios.

A. System Model

Consider the Phase Locked Loop system given in Fig. 1 which is also exposed to the interference signal $J(t)$ as well as additive noise $n(t)$ which is assumed to be a white, Gaussian stationary process with zero mean and single sided spectral density of height N_0 W/Hz and the desired carrier signal $s(t)$ which can be represented as

$$s(t) \quad = \quad \sqrt{2S}\,\sin\Phi(t) = \sqrt{2S}\,\sin(\omega_0 t + \Omega_0 t + \theta_0) \tag{1}$$

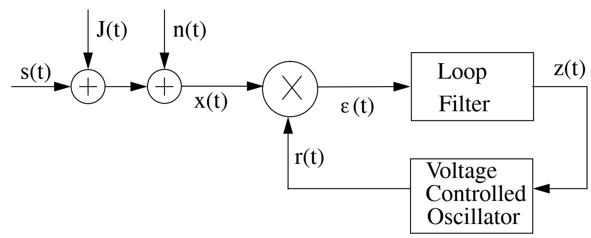

Fig. 1. Phase Locked Loop System Model

Here, the carrier is assumed to be of constant frequency and phase offsets (Ω_0 and θ_0) relative to the quiescent VCO oscillation of frequency, ω_0. We assume that the interference signal is a collection of $L+1$ CW signals

$$J(t) = \sqrt{2J_s}\,\sin(\omega_0 t + \Omega_s t + \theta_s) + \sum_{i=1}^{L}\sqrt{2J_i}\,\sin(\omega_0 t + \Omega_i t + \theta_i) \tag{2}$$

The first term on the right hand side of (2) is assumed to have specular nature which has constant power of J_s and is of constant frequency and phase offsets Ω_s and θ_s relative to the quiescent VCO frequency and phase. The remaining L sinusoids are also of constant power, J_i, and of frequency offsets Ω_i and random phase offsets θ_i $\quad i = 1.........L$ relative to the quiescent VCO. The effects of these terms on the loop performance will be characterized statistically.

With $s(t)$ and $J(t)$ as defined in (1) and (2), the equation that governs the phase error between the VCO output and the desired carrier ($\triangleq \varphi(t)$) can be shown to satisfy [18]

$$\frac{1}{K\sqrt{S}}\frac{d\varphi(t)}{dt} = \gamma - F(p)\left[\sin\varphi + \sqrt{R_s}\sin(\varphi + \Delta\Omega\, t + \Delta\theta) + I_r(\varphi,t) + \frac{1}{\sqrt{S}}N(t)\right] \tag{3}$$

$$I_r(\varphi,t) \triangleq \sum_{i=1}^{L}\sqrt{R_i}\,\sin(\varphi + \Delta\Omega_i t + \Delta\theta_i) \tag{4}$$

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$$\gamma \triangleq \frac{\Omega_o}{K\sqrt{S}}, \qquad R_s \triangleq \frac{J_s}{S}. \qquad R_i \triangleq \frac{J_i}{S} \quad i = 1.......L \tag{5}$$

where, K is the open loop gain, $\Delta\Omega \triangleq \Omega_s - \Omega_0$ and $\Delta\theta \triangleq \theta_s - \theta_0$ are the frequency and phase offsets between the specular interferer and the desired carrier, $\Delta\Omega_i \triangleq \Omega_i - \Omega_0$ and $\Delta\theta_i \triangleq \theta_i - \theta_0$ are the frequency and phase offsets between the i^{th} random interferer and the desired carrier,. $F(p)$ is the loop filter characterized in Heaviside operator notation ($p \triangleq \frac{d}{dt}$ is the Heaviside operator). $N(t)$ is a white Gaussian Noise Process with single sided spectral height N_0 W/Hz.

The total interference power to signal power ratio with which the loop is exposed ($\triangleq R_{eq}$) in the presence of multiple interferers is defined as

$$R_{eq} \triangleq R_s + R_r = R_s + \sum_{i=1}^{L} R_i \tag{6}$$

where R_r is the total random multipath interference power to desired signal power ratio.

B. Statistical Characterization of Random Interference Effects on the Loop Equation

It is possible to write the random interference components represented by $I_r(t,\varphi)$ in the loop equation (3) as

$$I_r(\varphi,t) = I_L(t)\sin\varphi \; + \; Q_L(t)\cos\varphi \tag{7}$$

$$I_L(t) \triangleq \sum_{i=1}^{L} \sqrt{R_i}\, \cos(\Delta\Omega_i\, t + \Delta\theta_i) \tag{8}$$

$$Q_L(t) \triangleq \sum_{i=1}^{L} \sqrt{R_i}\, \sin(\Delta\Omega_i\, t + \Delta\theta_i) \tag{9}$$

We assume that the phases of the interferer tones, $\theta_i \quad i = 1.....L$ are independent and uniformly distributed random variables in $[0, 2\pi]$. Then $\Delta\theta_i \quad i = 1.....L$ are independent and uniformly distributed and $I_L(t)$ and $Q_L(t)$ are wide sense stationary processes with autocorrelation function

$$R_{I_L}(\tau) = R_{Q_L}(\tau) = \frac{1}{2}\sum_{i=1}^{L} R_i\, \cos(\Delta\Omega_i\, \tau) \tag{10}$$

The power density spectrum related to (10) is given as

$$S_{I_L}(\omega) = S_{Q_L}(\omega) = \frac{1}{4}\sum_{i=1}^{L} R_i[\delta(\omega - \Delta\Omega_i) + \delta(\omega + \Delta\Omega_i)] \tag{11}$$

We assume that $S_{I_L}(\omega)$ is narrow band relative to the loop bandwidth. Then the variations in force terms in (3) are much slower relative to the loop time constants. Thus, for short term analysis of the loop $\Delta\theta' \triangleq \Delta\Omega_i\, t_0 + \Delta\theta \quad i = 1.....L$ can be assumed constant for arbitrary time instant t_0. Then time dependence on (4), (8) and (9) can be dropped. Furthermore $\Delta\theta_i' \quad i = 1.....L$ are independent and uniformly distributed in $[0, 2\pi]$. Note that for the multipath scenario in which the multiple reflections of the carrier tone reaches the receiver without Doppler shift ($\Delta\Omega_i = 0 \quad i = 1.....L$), $\Delta\theta_i' = \Delta\theta_i$ and no approximation is made.

Then, $I_L \triangleq \sum_{i=1}^{L}\sqrt{R_i}\, \cos\Delta\theta_i'$ and $Q_L \triangleq \sum_{i=1}^{L}\sqrt{R_i}\, \sin\Delta\theta_i'$ are zero mean uncorrelated random variables of equal variance $\sigma_{I_L}^2 = \sigma_{Q_L}^2 = R_r/2$.

Several authors have attempted to obtain statistical characterization of the forms given in (8) and (9) [20], [21], [22], [23], [24]. A common conclusion is that, as the the number of contributors, L , is increased, statistics of I_L and Q_L will approach Normal distribution [20], [21], [22], [23]. In [22] is concluded that for $L > 10$, the amplitude distribution of sum of equal magnitude sine waves approaches Normal distribution. This result can also be stated based on Central Limit Theorem arguments. We will assume that L is such large that the Gaussian approximation holds. Let $I \triangleq \lim_{L\to\infty} I_L$ and $Q \triangleq \lim_{L\to\infty} Q_L$, It can easily be shown that I and Q are jointly Gaussian as well as being marginally Gaussian. In summary, I and Q are Gaussian random variables with zero mean and variance $\sigma^2 = \sigma_I^2 = \sigma_Q^2 = R_r\ /\ 2$. Furthermore, since I_L and Q_L are uncorrelated for any L, I and Q are uncorrelated, thus, independent random variables.

In the rest of this study we will assume that $\Delta\Omega = 0$ (i.e. the specular component and the desired carrier have the same frequency) which represents a worst case scenario. In the light of above discussions and by utilizing a trigonometric identity the loop equation (3) can be written as

$$\frac{1}{K\sqrt{S}}\frac{d\varphi(t|I,Q)}{dt} = \gamma - F(p)\left[M_2\sin(\varphi + P_2) + \frac{1}{\sqrt{S}}N(t)\right] \tag{12}$$

$$M_2 \triangleq \sqrt{(1 + I + \sqrt{R_s}\cos\Delta\theta)^2 + (\sqrt{R_s}\sin\Delta\theta + Q)^2}.$$

$$P_2 \triangleq \tan^{-1}\left[\frac{\sqrt{R_s}\sin\Delta\theta + Q}{1 + I + \sqrt{R_s}\cos\Delta\theta}\right]$$

II. ANALYSIS IN THE PRESENCE OF SPECULAR AND RANDOM INTERFERENCE

In the absence of noise, assuming that a stable lock point exists for the loop, since the steady state phase error is constant, by equating the left hand side of (12) to zero, the steady state phase error ($\triangleq \varphi_{ss}(I,Q)$) can be written as

$$\varphi_{ss}(I,Q) = \sin^{-1}\left[\frac{\gamma}{M_2\, F(0)}\right] - P_2 \; \pm \; 2\,n\,\pi. \tag{13}$$

Here n is any integer and $F(0)$ is the loop filter transfer function at the origin. The existence of a stable solution in the presence of detuning requires that

$$\left|\frac{\gamma}{M_2\, F(0)}\right| < 1 \tag{14}$$

It is always possible to get a stable solution for a perfect second order loop ($F(0) = \infty$); however, this is not the case for a for a first order or an imperfect second order loop ($F(0) = 1$) . We now obtain the lock condition for an arbitrary $\Delta\theta$. Since $\Delta\theta\ \epsilon\ [0, 2\pi]$ and it can easily be shown that the smallest value of the denominator in (14) is $|\sqrt{(1+I)^2 + Q^2} - \sqrt{R_s}|$ (it is attained when $\Delta\theta = -\tan^{-1}[(1+I)/Q]$). In order for (3) to have a stable solution for arbitrary $\Delta\theta$, one should have

$$\left|\frac{\gamma/F(0)}{\sqrt{(1+I)^2 + Q^2} - \sqrt{R_s}}\right| < 1 \tag{15}$$

For first order and imperfect second order loops, for a given specular interference scenario, due to marginally normal statistics of I and Q, theoretically there are always some realizations such that (14) is not satisfied and the loop is forced out of lock. In Fig. (2) the region corresponding to loss of lock is given on

Fig. 2. The locus of interference scenarios that forces the loop to lose lock (Shaded Region).

In the rest of the analysis in the absence of noise we assume that no frequency offset between the desired carrier and quiescent VCO frequency exists ($\gamma = 0$) for the analysis of first and imperfect second order loops. No assumption on γ is required for the analysis of perfect second order loops.

Consider the phasor respresentation given in Fig. 3 were the vector S represents the desired signal, D represents the specular interference component. I and Q represent the components of the interference signal, U, that are in-phase and quadrature with the desired signal respectively. The loop tracks the sum, R, of desired and interference vectors and a steady state phase measurement error, φ_{ss}, occurs.

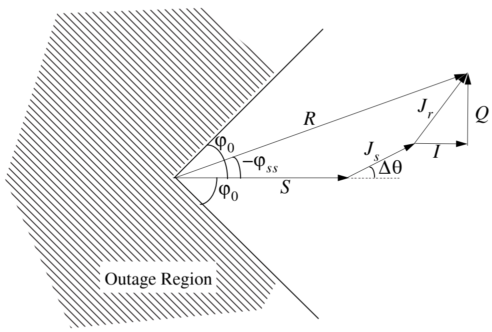

Fig. 3. Phasor Representation in the Presence of a Specular Path and Multiple Random Paths

The phase error has a probability density function given by [25]

$$p(\varphi_{ss}) = \frac{e^{-\frac{A^2}{2\sigma^2}}}{2\pi} + \frac{A\cos(\varphi_{ss}-\varphi_D)}{(2\pi)^{1/2}} \cdot e^{-\frac{A^2\sin^2(\varphi_{ss}-\varphi_D)}{2\sigma^2}} \cdot [1 - Q(\frac{A\cos(\varphi_{ss}-\varphi_D)}{\sigma})] \quad (17)$$

$$\varphi_D \triangleq -\tan^{-1}(\frac{\sqrt{R_s}\sin\Delta\theta}{1+\sqrt{R_s}\cos\Delta\theta})$$

$$A \triangleq \sqrt{1 + R_s + 2\sqrt{R_s}\cos(\Delta\theta)}$$

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\, dt$$

In Figs. 4 and 5 we plot the mean $m(\varphi_{ss})$, and the standard deviation $\sigma(\varphi_{ss})$, vs R_r for for several specular component scenarios respectively.

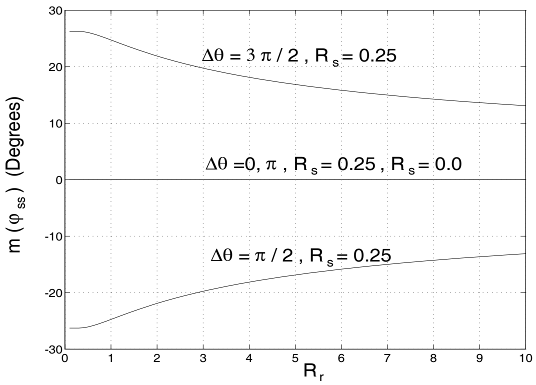

Fig. 4. Mean, $m(\varphi_{ss})$ vs. R_r for Several Interference Scenarios.

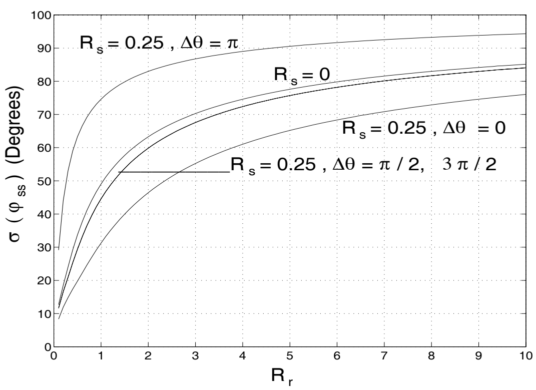

Fig. 5. $\sigma(\varphi_{ss})$ vs. R_r for Several Interference Scenarios.

Depending on the values I and Q, the steady state phase error may be quite large although the loop remains locked to the desired carrier (with phase error given as in (13)). Thus, it is necessary to establish a criterion for satifactory operation in terms of phase error magnitude. We assume that the PLL is rendered in outage if $|\varphi_{ss}| > \varphi_0$, where φ_0 is a pre-chosen value suitable with the application and performance requirements of the system (see Fig. 3). The PLL outage probability ($\triangleq P(outage)$) can then be calculated through

$$P(outage) = \int_{\varphi_0}^{2\pi-\varphi_0} p(\varphi_{ss})\, d\varphi_{ss} \quad (18)$$

The outage probability of the loop may be expressed as [26]

$$P(outage) = \frac{1}{2\pi}\int_0^{\pi-(\varphi_0+\varphi_D)} e^{-\frac{A^2\sin^2(\varphi_0+\varphi_D)}{2\sigma^2\sin^2\varphi_{ss}}}\, d\varphi_{ss} + \frac{1}{2\pi}\int_0^{\pi-(\varphi_0-\varphi_D)} e^{-\frac{A^2\sin^2(\varphi_0+\varphi_D)}{2\sigma^2\sin^2\varphi_{ss}}}\, d\varphi_{ss} \quad (19)$$

In Fig. 6 $P(outage)$ is depicted as a function of the total interference to signal ratio, R_r for $\varphi_0 = 45^o$.

III. ANALYSIS IN THE PRESENCE OF NOISE

Equations of the type (12) has been studied extensively in [13]. In particular, (12) represents the equation of operation

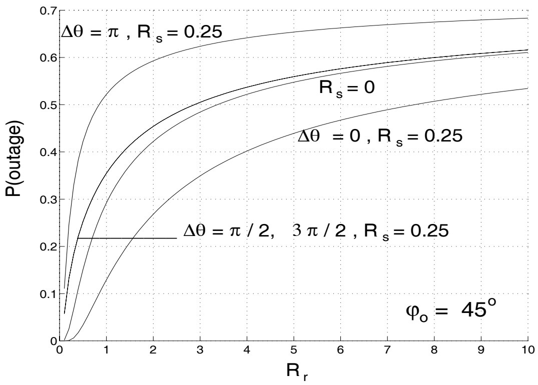

Fig. 6. $P(outage)$ vs. R_r for Several Interference Scenarios.

of a second order tracking loop with phase detector characteristics $g(\varphi) = M_2 \sin(\varphi + P_2)$. Results of [13] can be utilized to yield the steady state probability density function $p(\phi)$ of the modulo-2π reduced phase error $\phi(t)$ for the second order loop

$$p(\phi|I,Q) = \frac{1}{4\pi^2 \;\; e^{-\beta_1 \pi} \;\; |I_{j\beta_1}(\alpha_1)|^2} \, e^{\beta_1 \phi + \alpha_1 \cos(\phi + P_2)} \cdot \int_{\phi}^{\phi + 2\pi} e^{-[\beta_1 y + \alpha_1 \cos(y + P_2)]} \, dy. \qquad (20)$$

$$\beta_1 \triangleq \left(\frac{rM_2 + 1}{rM_2}\right) \frac{\rho' M_2^2}{F_0} \left[\frac{\gamma}{M_2} - (1 - F_0)\,\overline{\sin(\phi + P_2)}\right] \cdot \left[1 + \frac{F_0}{\rho' M_2^2 (rM_2 + 1)\sigma_G^2}\right]$$

$$\alpha_1 \triangleq \left(\frac{rM_2 + 1}{rM_2}\right) \rho' M_2^2 - \frac{1}{rM_2 \sigma_G^2}$$

$$F_0 \triangleq T_2/T_1 \qquad r \triangleq F_0 T_2 K \sqrt{S}$$

$$\rho' \triangleq \frac{S}{N_0 B_L'} \qquad B_L' \triangleq \frac{rM_2 + 1}{4T_2}$$

$$\sigma_G^2 \triangleq \overline{\sin^2(\phi(t) + P_2)} - \overline{\sin(\phi(t) + P_2)}^2$$

Here $I_{j\beta_1}(\cdot)$ is Modified Bessel Function of purely imaginary order $j\beta_1$, and, overbar denotes statistical expectation. The probability density function for a first order loop follows from (20) by setting $F_0 = 1$ and letting $r \to \infty$.

The unconditioned phase error probability density function of the phase error can be found by averaging over the normal densities of the in-phase and quadrature terms I and Q. In Figs. 7 and 8 we present $p(\phi)$ for a first order loop; Absence of specular interference component ($R_s = 0$) is assumed and the results are given for several interfernce to desired signal power ratio R_r values, for $\gamma = 0$ and $\gamma = 0.5$ respectively. Also the pdf in the absence of interferers is presented in these figures.

In Figure 9 the mean of the phase error is plotted as a function of R_r for $\gamma = 0.5, R_s = 0$ for a first order loop. If $\gamma = 0$ and $R_s = 0$, $p(\phi)$ has zero mean. This can be seen from the symmetry of (20) around its mean and the symmetry of the density functions of the I and Q.

In Fig. 10 the variation of standard deviation in the absence of the specular component as a function of R_r is depicted for $\gamma = 0$ and $\gamma = 0.5$ for a first order loop. Also the standard deviation in the absence of interferers can be observed from these figures.

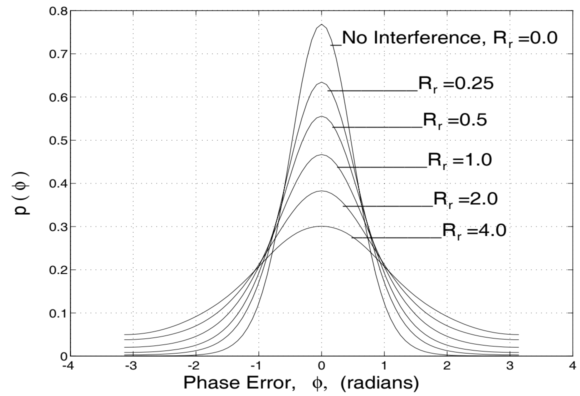

Fig. 7. Phase Error Density Function $p(\phi)$ in the Presence of Multiple Random Interferers ($\alpha_1 = 4, \gamma = 0, R_s = 0, \; F(p) = 1$).

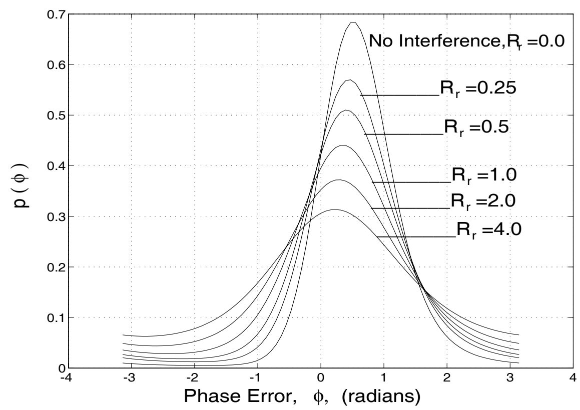

Fig. 8. Phase Error Density Function $p(\phi)$ in the Presence of Multiple Random Interferers ($\alpha_1 = 4, \gamma = 0.5, R_s = 0, \; F(p) = 1$).

Conditioned on I and Q, the probability current related to $\phi(t)$ ($\triangleq J_{\phi|I,Q}$) and the cycle slip rates in the positive and negative directions ($N^+_{\phi|I,Q}$ and $N^-_{\phi|I,Q}$ respectively) for a first order loop can be obtained by using standard techniques to be [18]

$$J_{\phi|I,Q} \;=\; [N^+_{\phi|I,Q} - N^-_{\phi|I,Q}] = \frac{\Omega_0}{2\pi} \; \frac{\mathrm{sinch}(\beta_1 \pi)}{|I_{j\beta_1}(M_2)|^2} \qquad (21)$$

$$N^{\pm}_{\phi|I,Q} = K\sqrt{S} \; \frac{1}{4\pi^2 \alpha_1} \, \frac{e^{\mp \pi \beta_1}}{|I_{j\beta_1}(M_2)|^2}. \qquad (22)$$

Here, $\mathrm{sinch}(x) \triangleq \sinh(x)/x$. If one desires to obtain the unconditioned dynamics of $\phi(t)$, it is necessary to perform the averaging of quantities of interest over the distributions of I and Q. Denoting the slip rates in the positive and negative directions in the absence of interferers with N_a^+ and N_a^- respectively the effect of interference can then be examined by the ratios:

$$E_{I,Q} \;\triangleq\; \frac{N^+_{\phi|I,Q}}{N_a^+} \;=\; \frac{N^-_{\phi|I,Q}}{N_a^-} \;=\; \frac{|I_{j\beta_1}(\alpha_1)|^2}{|I_{j\beta_1}(M_2)|^2}. \qquad (23)$$

Here "$E_{I,Q}$" denotes the increase factor in the slip rates by the introduction of the interferers. The average increase factor, E, can be obtained by averaging (23) over $p(I)$ and $p(Q)$.

IV. **CONCLUSIONS**

We introduced a statistical characterization of effects of multiple CW interferers on the steady state operation performance of Phase Locked Loops. It has been found that first

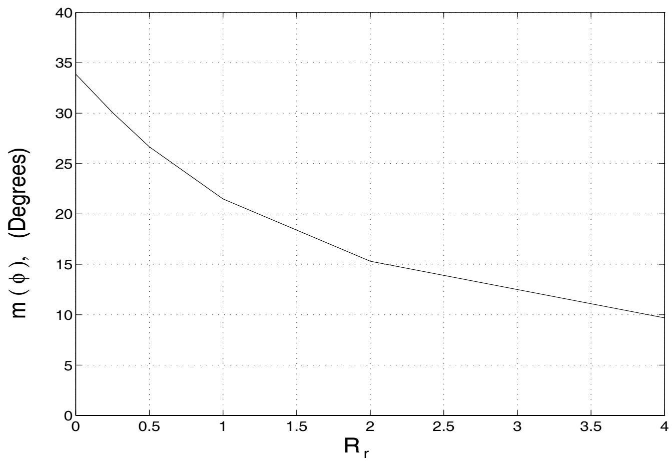

Fig. 9. Mean, $m(\phi)$ vs R_r In the Presence of Multiple Random Interferers ($\alpha_1 = 4, \gamma = 0.5, R_s = 0,\ F(p) = 1$).

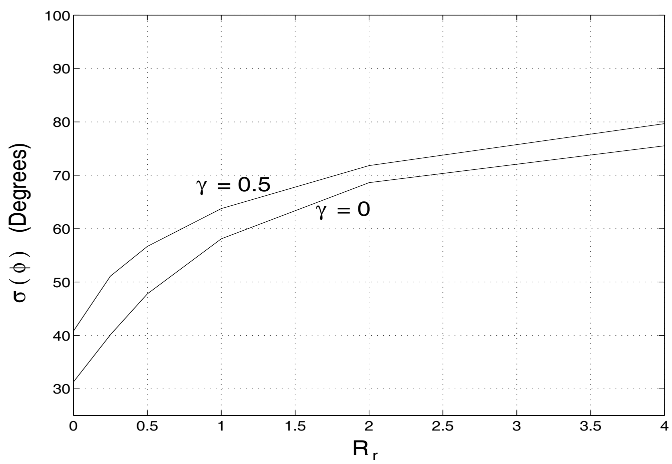

Fig. 10. Standard Deviation, $\sigma(\phi)$ vs R_r In the Presence of Multiple Random Interferers ($\alpha_1 = 4, R_s = 0,\ F(p) = 1$).

and imperfect second order loops are susceptible with respect to losing lock in the presence of interference when there is initial detuning between the desired carrier and the idle VCO frequencies. Perfect second order loops do not display this vulnerability. The outage probabilities that are obtained through a novel definition for outage shows that in the presence of a specular interferer, the performance degradation depends on the the relative phases of the desired carrier and the specular interferer.

Results in the presence of noise reveal that conditioned on the the in-phase and quadrature components of the random interference, loop SNR is modified and a bias is added to the phase error process. In absence of specular interference, averaging over the statistics of interference reveals a spreading of the phase error probability density function, thus an increase in the variance of the phase error.

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