Effects of CW Interference on Carrier Tracking *

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Abstract

This paper focuses on the modeling and analysis of phase locked loops in the presence of CW interference such that the operating vulnerability to CW jamming and interference can be accessed. In the absence of noise the loop phase error signal is characterized analytically and the conditions under which the loop remains locked to the desired carrier are presented. Analysis has been conducted for arbitrary frequency detuning between the desired carrier and the quiescent Voltage Controlled Oscillator (VCO) frequencies and arbitrary frequency offsets between the interferer and desired signals. The results show that loop performance degradation depends on the frequencies of the interferer and the desired signal relative to the center VCO frequency; it also depends upon the interference power to the desired signal power ratio. In the presence of noise and interference, a model which manifests the time evolution of the phase error process is developed. A Fokker Planck analysis is conducted on this model in order to illustrate the method and obtain insight into system performance in the presence of interferer.

1 Introduction

Synchronization of the locally generated reference phase with the incoming carrier phase is a fundamental phase of demodulation process in a coherent communications receiver. In many circumstances, carrier synchronization system performance is degraded by both additive noise and interferences. Due to crowding of the useful frequency spectrum many systems are now being effected by interferences more than they were a few decades ago. Interferences may also be jamming signals from unfriendly sources.

Application areas of the results and the developed tools of this study includes CW interference effects on telemetry/range-Doppler measurements (for example NASA's Deep Space Network (DSN) [1, 2]); Coherent monopulse angle tracking radar performance analysis in the presence of a secondary target within the main lobe of the transmitter antenna (which leads to sinusoidal signature) [3, 4, 5]; Spurious frequencies leaking from the local oscillator circuitry in a receiver.

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The performance analysis of carrier synchronization systems has been the subject of several studies. Majority of these assume the absence of additive noise [2, 3, 4, 5, 6, 7, 9, 10, 15, 16]. If Signal to Noise Ratio (SNR) that effects the synchronization circuitry is such high that the primary agent effecting the system is the interferer, deterministic approach gives accurate results about system behavior.

One technique that has been utilized is the computer simulation by implementing the loop equation in a computer code and observing the phase error [6, 7, 8]. In [6] and [7] the critical interference to signal power ratio (ISR) beyond which the loop loses lock is presented. The study is conducted by slowly increasing the amplitude of the interferer and observing the trajectory of the phase error. Beyond a certain critical value, the phase error is no longer periodic but increases without bound. In [8], performance of a perfect second order loop is examined by computer simulations in a noisy, specular plus diffuse multipath environment. The standard deviation of the phase jitter is used as a criterion to yield performance curves.

Analytical studies of the references [2, 3, 4, 5, 9, 10, 16] utilize harmonic balance method either directly or indirectly in order to analyze the dynamics of the phase error signal in the absence of noise. Due to time periodic force term in the loop equation due to CW interferer, the phase error is periodic and thus it can represented in the form of a Cosine series. Approximation of such expansion by d.c. and first harmonic terms leads to the following model [2, 9, 10]

$$\varphi(t) = c_o + c_1 \cos(\Delta\Omega\, t + \Delta\theta + \psi_1). \qquad (1)$$

For determining $\varphi(t)$ one can insert (1) into (2) and use "Harmonic Balance Method" to obtain c_0 , c_1 and ψ_1. The method involves, as the name implies, equating the relevant coefficients of left and right hand sides of the equation for each harmonic. Efforts to incorporate higher order harmonics in the Harmonic Balance Method leads to untractable sets of equations. The validity of this model substantiated over a large range of desired carrier, interference and loop parameters, and, is also supported by experimental observation [9].

Figure 1: Carrier Synchronization System Model

The equation governing the phase error process $\varphi(t)$ can be shown to satisfy:

$$\frac{1}{K\sqrt{S}}\,\frac{d\varphi(t)}{dt} = \gamma - F(p)[\sin\varphi(t) + \sqrt{R}\,\sin(\varphi(t) + \Delta\Omega t + \Delta\theta) + \frac{1}{\sqrt{S}}N(t)] \quad (2)$$

with

$$\gamma \triangleq \frac{\Omega_o}{K\sqrt{S}}, \qquad R \triangleq \frac{J}{S}. \quad (3)$$

Here, R is the interference power to the signal power ratio, K is the open loop gain, $\Delta\Omega \triangleq \Omega_i - \Omega_0$ and $\Delta\theta \triangleq \theta_i - \theta_0$ are the frequency and phase offsets between the interferer and the desired carrier respectively, and, $F(p)$ is the loop filter characterized in Heaviside operator notation ($p \triangleq d/dt$ is the Heaviside operator). For a first order loop, the loop filter is identified by $F(p) = 1$, for a perfect second order loop $F(p) = (1 + pT_2)/\,pT_1$, and, for an imperfect second order loop $F(p) = (1 + pT_2)/(1 + pT_1)$. For future usage, let us introduce the notation $M(\omega) = |F(\omega)|$, and, $P(\omega) = \angle(F(\omega))$ (i.e. $M(\omega)$ and $P(\omega)$ are magnitude and phase response characteristics of the loop filter $F(\omega)$). The noise process, $N(t)$, appearing (2) is a stationary, white Gaussian noise zero mean and two sided spectral density of height $N_0/2$ W/Hz [13].

3 Loop Performance In the Absence of Noise

By utilizing the model given in (1), we employ Harmonic Balance Method. After a few suitable approximations the magnitude of the oscillations (c_1) and the average phase error (c_0) in the presence of CW interferer can be shown be [19]:

$$c_1^2 = \frac{R}{D^2\left[\left(\frac{\cos P(\Delta\Omega)}{M(\Delta\Omega)}\right)^2 + \left(\frac{\sin P(\Delta\Omega)}{M(\Delta\Omega)} + \frac{\sqrt{1-\gamma^2}}{D}\right)^2\right]}, \quad (4)$$

$$\sin c_0 \quad = \quad \frac{\gamma}{M(0)} - \frac{{c_1}^2 D}{2}\frac{\cos P(\Delta\Omega)}{M(\Delta\Omega)}. \quad (5)$$

Here $D \triangleq \Delta\Omega\,/\,K\sqrt{S}$ is the normalized detuning between the desired carrier and interference. These results are accurate for $|\Delta\Omega| > 4B_L$ where B_L is the one sided linearized loop bandwidth.

The condition on loop, desired signal and interference parameters which will quarantee locking onto the desired carrier can be derived by invoking a periodicity condition on the phase error signal. The critical value R_{cr} of interference to signal power ratio beyond which the loop loses its lock to the desired signal is:

$$R_{cr} \;=\; \left(\frac{\gamma}{M(0)} + \mathrm{sgn}(\Delta\Omega)\right)\,\frac{2D}{M(\Delta\Omega)\cos P(\Delta\Omega)}. \quad (6)$$

For the case $\gamma = 0$, these results reduce to the earlier results [2, 9]. We present R_{cr} vs $\Delta\Omega/B_L$ for first, perfect second order and imperfect second order loops in Fig. 2 for $\gamma = 0.0$ and $\gamma = 0.5$. From Fig. 2, it can be seen that the first order loop is highly inferior to second order loops with respect to holding its lock to the desired carrier. For $\gamma = 0$, the perfect and the imperfect loops act similarly. It is seen that the performance of the perfect second order loop is not affected by initial detuning in the loop. This can also be seen by examination of (6) by observing the presence of the loop filter pole at origin ($M(0) = \infty$). However, imperfect second order loop and the first order loop are highly vulnerable to initial detuning. For the given positive detuning parameter value of $\gamma = 0.5$, it is seen that the loop is more vulnerable to CW effects for negative values of $\Delta\Omega$ rather than positive values of $\Delta\Omega$.

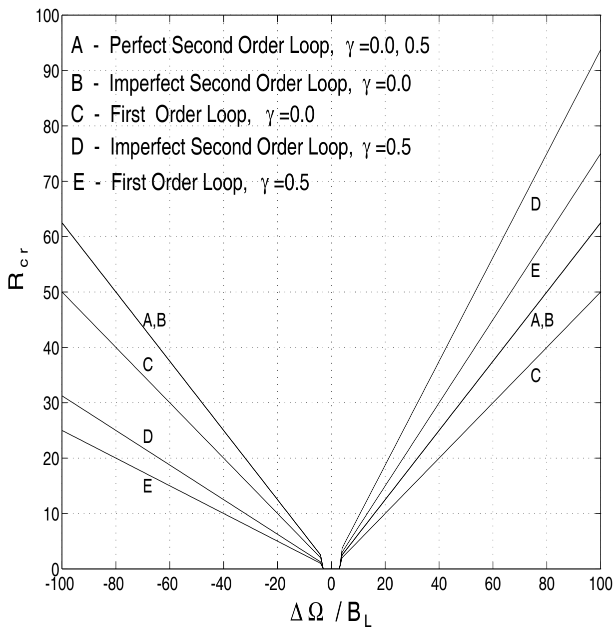

Figure 2: R_{cr} vs. $\Delta\Omega/B_L$ for First, Perfect, and, Imperfect Second Order (critically damped) Loops ($T_2/T_1 = 0.025,\ \ \gamma = 0.0, 0.5$).

When $|\Delta\Omega| < 4B_L$, the model given in (1) loses its accuracy due to significant higher harmonics. Thus, the above results are not applicable. For this case, suitable techniques of analysis are discussed in [19].

4 Loop Performance In the Presence of Noise

For statistical characterization of the loop behavior in the presence of noise one may try to use Fokker Planck techniques. However, it is not possible to get a closed-form analytical solution to the Fokker-Planck equation related to (2) [20]. Here we develop an approximate model for the evolution of the phase error process utilizing the phase error trajectory in the absence of noise. The technique is similar to Krylov-Boguliubov method for analysis of underdamped sinusoidal oscillators [21, 22]. In this model the phase error process $\varphi(t)$ is written as:

$$\varphi(t) = \tilde{z}(t) + c_1 \cos(\psi_1 + \Delta\Omega t + \Delta\theta); \qquad |\Delta\Omega| > 4B_L \tag{7}$$

Here, the second term is the periodic beatnote in the phase error trajectory that is characterized in the absence of noise by Harmonic Balance Method. The first term $\tilde{z}(t)$ is called the **time averaged phase error process** since its dynamics is described by the Langevin equation whose force term is obtained by averaging the time varying force term of the Langevin equation describing $\varphi(t)$

$$\frac{1}{K\sqrt{S}}\dot{\tilde{z}} = \gamma - F(p)\left\{\left[J_0(c_1) + \sqrt{R}J_1(c_1)\ \sin\psi_1\right]\sin\tilde{z} + \sqrt{R}J_1(c_1)\cos\psi_1\cos\tilde{z} + \frac{1}{\sqrt{S}}N(t)\right\} \tag{8}$$

Here $J_o(\cdot)$ and $J_1(\cdot)$ are Bessel functions of order 0 and 1 respectively. Equation (8) is a Langevin Equation with time independent coefficients. Thus, it can be analyzed by standard Fokker Planck techniques to yield the steady state probability density function (pdf). We now examine the effects of the presence of CW interferer for a first order loop. However it should be noted that the method is applicable to loops of arbitrary order. For a first order PLL the probability density function of the modulo-2π reduced version of $\tilde{z}(t)$ which we call $z(t)$ is given by [19]:

$$p(z) = \frac{1}{4\pi^2\ e^{-\beta\pi}\ |I_{j\beta}(M_1)|^2}\ e^{\beta z + A_1\cos z - A_2 \sin z} \cdot \int_z^{z+2\pi} e^{-(\beta y + A_1 \cos y - A_2 \sin y)}\ dy \tag{9}$$

Here, the loop signal to noise ratio, α , and loop detuning, β, and, the remaining parameters of (9) are defined as:

$$\alpha \triangleq \frac{4\sqrt{S}}{N_0 K} \qquad \beta = \gamma\alpha \tag{10}$$

$$A_1 \triangleq \alpha\left[J_0(c_1) + \sqrt{R}J_1(c_1)\ \sin\psi_1\right] \tag{11}$$

$$A_2 \triangleq \alpha\sqrt{R}\ J_1(c_1)\ \cos\psi_1 \tag{12}$$

$$M_1 \triangleq \sqrt{A_1^2 + A_2^2}. \tag{13}$$

In Fig. 3, we present $p(z)$ for several interference scenarios for the case of $\gamma = 0.5$. Relative to the case of absence of interference ($R = 0$) it is seen that when an interferer is introduced with $R = 1$, $z(t)$ has larger bias and variance when $D = -2$ compared to the case of $D = 2$. Thus, along with the magnitude of D, its sign is important in performance analysis for nonzero γ. In Figs. 4 and 5, the mean and the standard deviation of $z(t)$ as a function of interference to desired carrier magnitude ratio, R, are depicted for several values of D. In the incomplete portions of these figures the loop is no longer locked to the desired carrier.

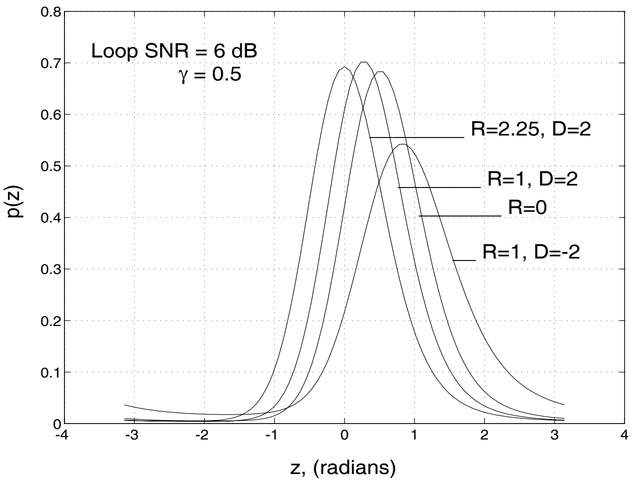

Figure 3: Steady State pdf of Time Averaged Phase Error Process $z(t)$ for Various Interference Scenarios. ($\alpha = 4, \beta = 2$) .

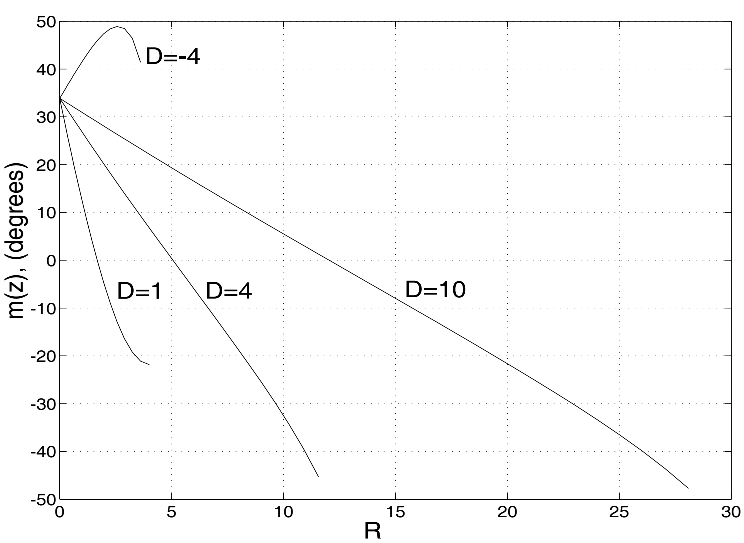

Figure 4: Mean $m(z)$ versus R for Various Values of D ($\beta = 2, \alpha = 4$)

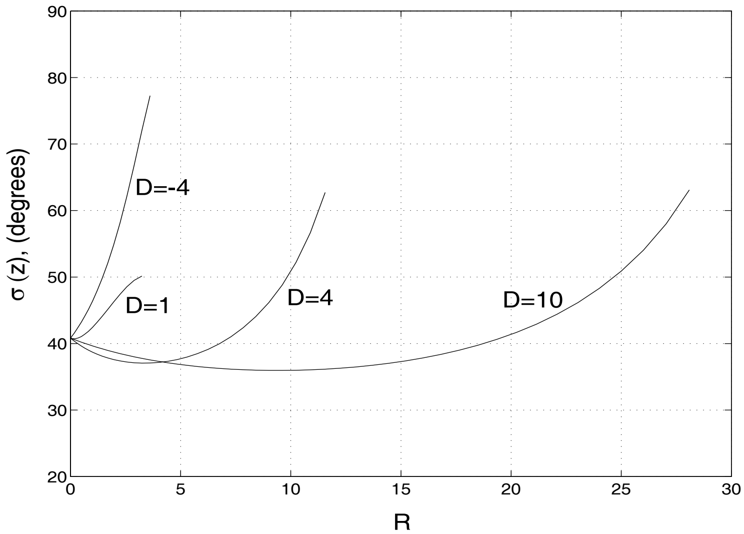

Figure 5: Standard Deviation $\sigma(z)$ versus R for Various Values of D. ($\beta = 2, \alpha = 4$)

The probability current related to $z(t)$, (J_z), and cycle slip rates in the positive and negative z directions (N_z^+ and N_z^-) are given by:

$$J_z = \left[N_z^+ - N_z^-\right] = \frac{\Omega_0}{2\pi} \, \frac{\text{sinch}(\beta\pi)}{|I_{j\beta}(M_1)|^2} \tag{14}$$

$$N_z^{\overset{+}{-}} = K\sqrt{S}\frac{1}{4\pi^2\alpha}\frac{e^{\overset{+}{-}\pi\beta}}{|I_{j\beta}(M_1)|^2}. \tag{15}$$

It is shown in [19] that these results hold for probability current and cycle slip rates for $\varphi(t)$ due to modeling of the phase error given in (7). We name the slip rates in positive and negative φ directions as N_φ^+ and N_φ^-. Denoting the slip rates in the absence of interference as N_a^+ and N_a^-, ehe effect of interference can be examined by the ratios:

$$E \triangleq \frac{N_\varphi^+}{N_a^+} = \frac{N_\varphi^-}{N_a^-} = \frac{|I_{j\beta}(\alpha)|^2}{|I_{j\beta}(M_1)|^2}. \tag{16}$$

Here the term "E" denotes the increase factor in the slip rates by the introduction of the interference. In Fig. 6 E vs R for various D values. enhancement factor for various interference scenarios are presented.

5 Conclusions

The initial detuning between the desired carrier and the quiescent VCO frequencies has been found to be imperative on PLL performance in the presence of CW interference if the loop filter does not have a pole at origin. If there is initial detuning, first order and imperfect second order PLL performance depends on the

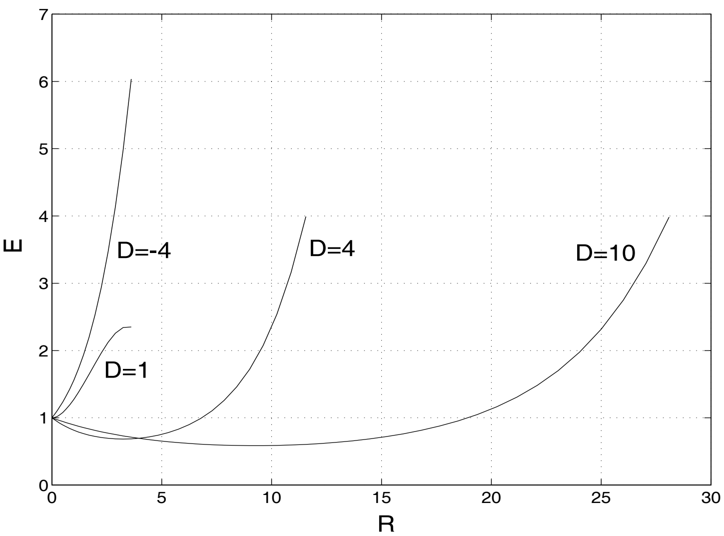

Figure 6: Enhancement Factor E versus R for Various Values of D ($\beta = 2, \alpha = 4$).

frequency difference between the desired carrier and interference signals not only in absolute value but also in sign. This dependence on relative spectral locations seems to not have been reported for the literature up to present time.

Further work is in progress for performance analysis of suppressed carrier loops in the presence of interference and effects of multiple interferers on carrier synchronization.

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